0606_11_Summer_2020_Q1

Solution

To find the expression for the **cubic curve** and solve the given inequality, we analyze the key features of the graph provided.

1. Expression for $f(x)$

The graph shows that the **x-intercepts** (roots) of the function are located at $x = -2$, $x = -1$, and $x = 5$. A cubic function with these roots can be written in the factored form:

$$f(x) = a(x - (-2))(x - (-1))(x - 5)$$

$$f(x) = a(x + 2)(x + 1)(x - 5)$$

The graph also identifies the **y-intercept** at the point $(0, 5)$. We substitute these coordinates into the equation to determine the leading coefficient a:

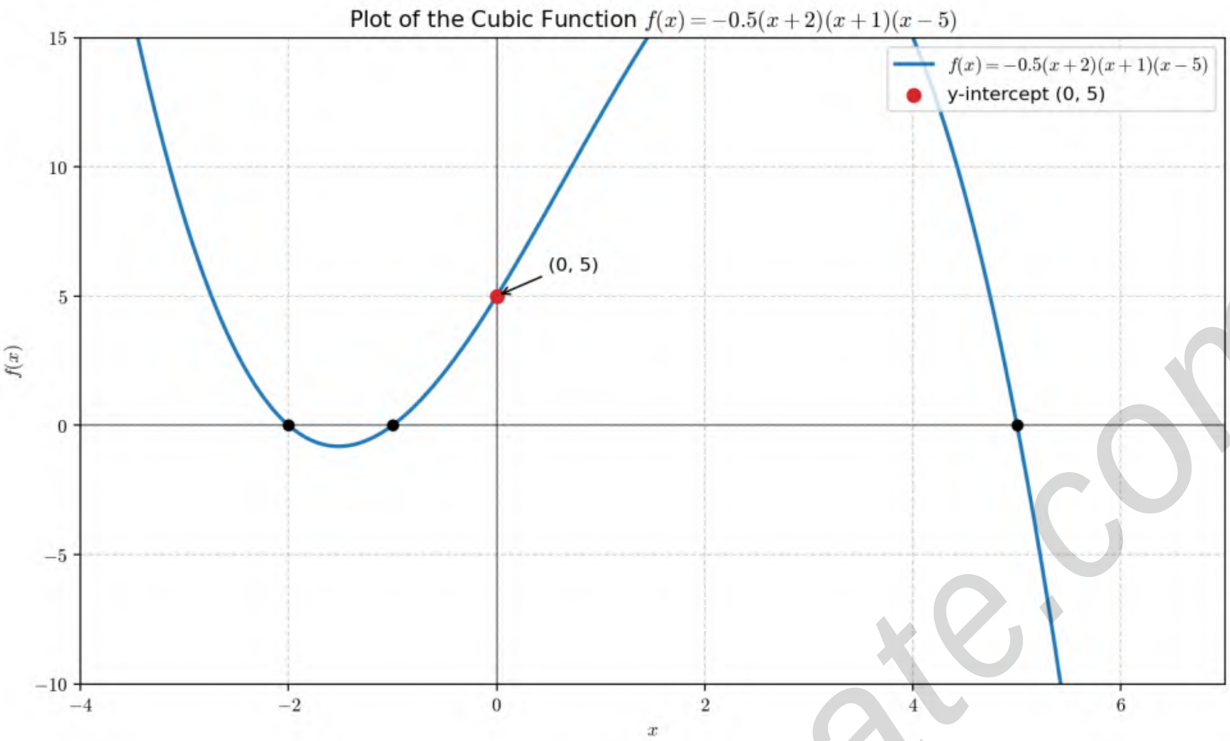
$$\begin{aligned} 5 &= a(0 + 2)(0 + 1)(0 - 5) \\ &= a(2)(1)(-5) \\ &= -10a \end{aligned}$$

Solving for a:

$$a = -\frac{5}{10} = -\frac{1}{2}$$

Substituting a back into the factored form, we obtain the expression for $f(x)$:

$$\boxed{f(x) = -\frac{1}{2}(x + 2)(x + 1)(x - 5)}$$



2. Solve $f(x) \leq 0$

The inequality $f(x) \leq 0$ is satisfied in the intervals where the graph is on or below the x-axis. By inspecting the graph:

- The curve is below the x-axis between the roots $x = -2$ and $x = -1$.
- The curve is below the x-axis for all values of x starting from the root $x = 5$ and extending to positive infinity.

Including the points where the function equals zero, the solution set is:

$$\boxed{-2 \leq x \leq -1 \quad \text{or} \quad x \geq 5}$$

0606_11_Summer_2020_Q2

Solution

To solve for the properties and the graph of the trigonometric function $y = 2\cos\frac{x}{3} - 1$, we analyze its standard form parameters.

1. Determining the Period

The general form of a **cosine function** is given by $y = a\cos(bx) + c$, where:

- a is the **amplitude**.
- b is the angular frequency coefficient.
- c is the vertical shift (the **midline**).

For the function $y = 2\cos\frac{x}{3} - 1$, we identify:

- $a = 2$
- $b = \frac{1}{3}$
- $c = -1$

The **period** T of the function (in degrees) is calculated using the formula:

$$\begin{aligned} T &= \frac{360^\circ}{\mid b \mid} \\ &= \frac{360^\circ}{1/3} \\ &= 1080^\circ \end{aligned}$$

The period of the function is $\boxed{1080^\circ}$.

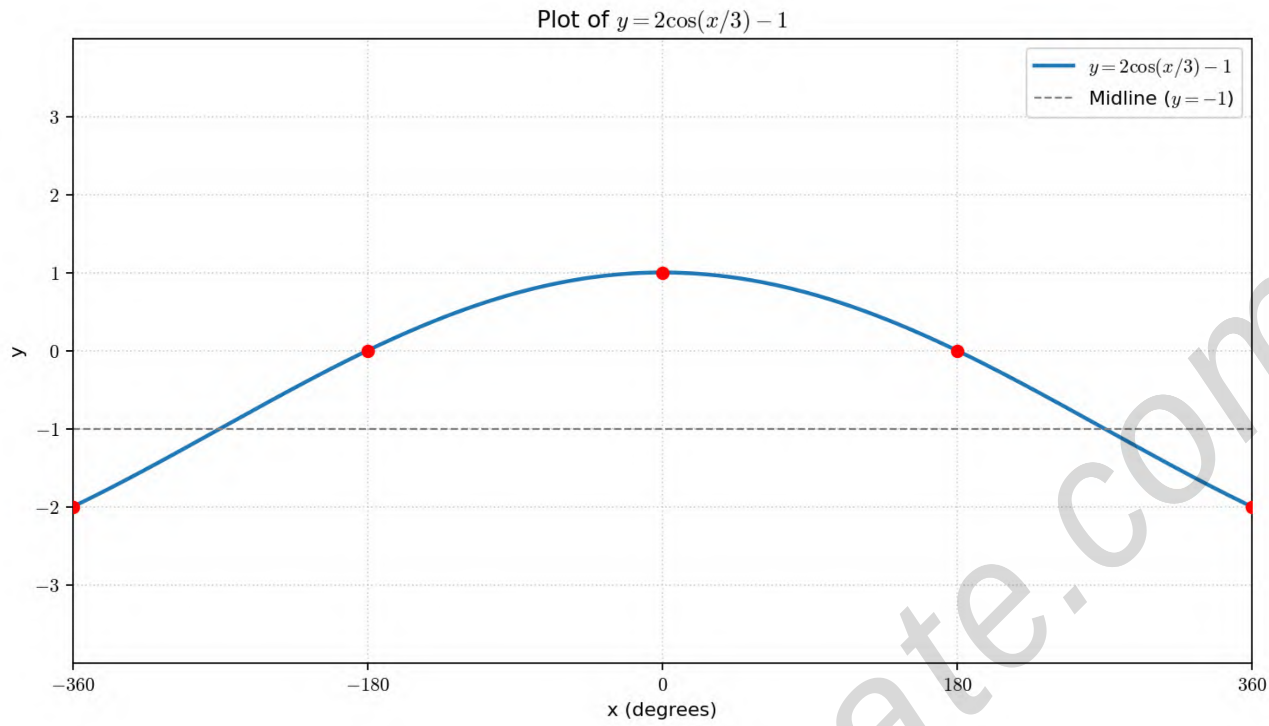
2. Sketching the Graph

To sketch the graph over the interval $-360^\circ \leq x \leq 360^\circ$, we first determine the key features:

- **Midline:** $y = -1$.
- **Maximum value:** $c + \mid a \mid = -1 + 2 = 1$.
- **Minimum value:** $c - \mid a \mid = -1 - 2 = -3$.
- **Y-intercept:** At $x = 0$, $y = 2\cos(0) - 1 = 2(1) - 1 = 1$.

Next, we calculate specific coordinates within the required domain:

- At $x = 180^\circ$: $y = 2\cos\left(\frac{180^\circ}{3}\right) - 1 = 2\cos(60^\circ) - 1 = 2(0.5) - 1 = 0$.
- At $x = 360^\circ$: $y = 2\cos\left(\frac{360^\circ}{3}\right) - 1 = 2\cos(120^\circ) - 1 = 2(-0.5) - 1 = -2$.
- At $x = -180^\circ$: $y = 2\cos(-60^\circ) - 1 = 2(0.5) - 1 = 0$.
- At $x = -360^\circ$: $y = 2\cos(-120^\circ) - 1 = 2(-0.5) - 1 = -2$.



The graph is a smooth curve passing through the points $(-360^\circ, -2)$, $(-180^\circ, 0)$, $(0^\circ, 1)$, $(180^\circ, 0)$, and $(360^\circ, -2)$.

0606_11_Summer_2020_Q3

Solution

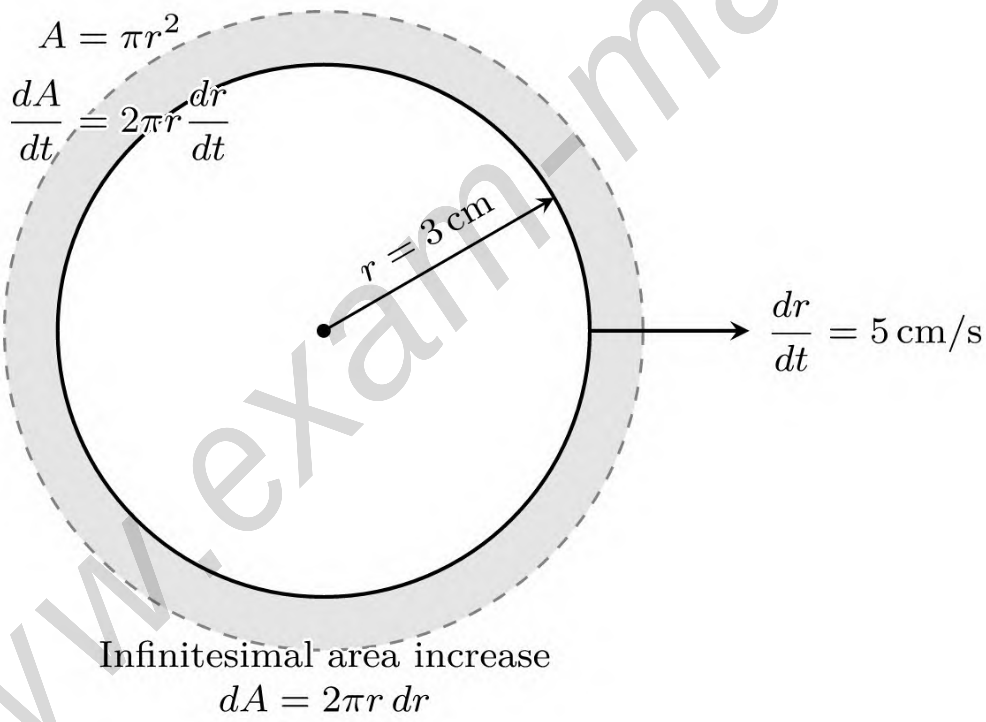
1. Identification of Given Parameters

The problem provides the instantaneous rate of change of the radius and the specific radius at which we need to evaluate the rate of change of the area. Let:

- r be the radius of the circle in cm.
- A be the area of the circle in cm^2.
- t be the time in s.

From the problem statement:

- The rate of change of the radius is $\frac{dr}{dt} = 5\,\text{cm}\cdot\text{s}^{-1}$.
- The radius at the instant of interest is $r = 3\,\text{cm}$.



2. Establishing the Geometric Relationship

The area A of a circle is related to its radius r by the standard geometric formula:

$$A = \pi r^2$$

3. Application of the Chain Rule****

To find the rate at which the area is increasing with respect to time, we differentiate the area formula with respect to t using the **related rates** method:

$$\begin{aligned}\frac{dA}{dt} &= \frac{d}{dt}(\pi r^2) \\ &= \frac{dA}{dr} \cdot \frac{dr}{dt} \\ &= 2\pi r \cdot \frac{dr}{dt}\end{aligned}$$

4. Calculation of the Rate of Change

Substituting the given values $r = 3$ and $\frac{dr}{dt} = 5$ into the derived expression:

$$\begin{aligned}\frac{dA}{dt} &= 2\pi(3)(5) \\ &= 30\pi\end{aligned}$$

The units for the rate of change of area are $\mathrm{cm}^2 \cdot \mathrm{s}^{-1}$.

$$\boxed{30\pi\,\mathrm{cm}^2 \cdot \mathrm{s}^{-1}}$$

0606_11_Summer_2020_Q4

Solution

To find the positive solution of the **quadratic equation** $(5+4\sqrt{7})x^2+(4-2\sqrt{7})x-1=0$, we apply the **quadratic formula**:

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

where the coefficients are:

- $A=5+4\sqrt{7}$
- $B=4-2\sqrt{7}$
- $C=-1$

1. Calculate the discriminant

The **discriminant** D is given by B^2-4AC:

$$\begin{aligned} D &= (4-2\sqrt{7})^2-4(5+4\sqrt{7})(-1) \\ &= (16-16\sqrt{7}+4\cdot 7)+4(5+4\sqrt{7}) \\ &= (16-16\sqrt{7}+28)+(20+16\sqrt{7}) \\ &= 44-16\sqrt{7}+20+16\sqrt{7} \\ &= 64 \end{aligned}$$

Since $D=64$, we have $\sqrt{D}=8$.

2. Determine the roots

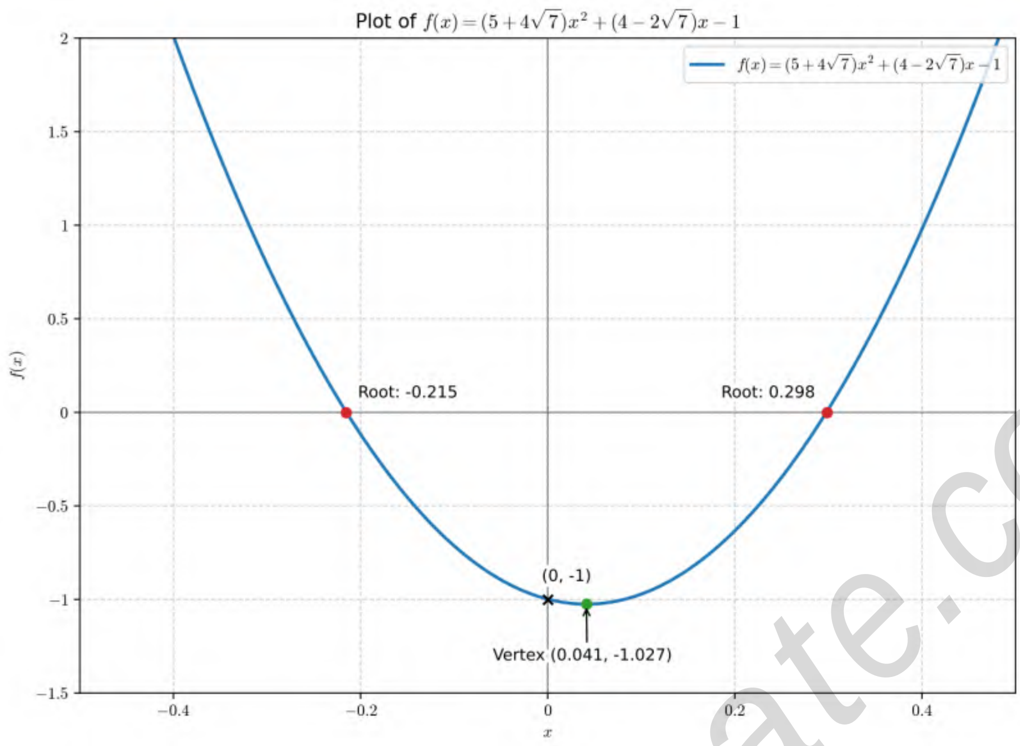
Substituting the values into the quadratic formula:

$$\begin{aligned} x &= \frac{-(4-2\sqrt{7})\pm 8}{2(5+4\sqrt{7})} \\ &= \frac{-4+2\sqrt{7}\pm 8}{10+8\sqrt{7}} \end{aligned}$$

This yields two possible solutions:

- $x_1=\frac{-4+2\sqrt{7}+8}{10+8\sqrt{7}}=\frac{4+2\sqrt{7}}{10+8\sqrt{7}}$
- $x_2=\frac{-4+2\sqrt{7}-8}{10+8\sqrt{7}}=\frac{-12+2\sqrt{7}}{10+8\sqrt{7}}$

Since $2\sqrt{7}=\sqrt{28}\approx 5.29$, the numerator of x_1 is positive $(4+5.29>0)$ and the numerator of x_2 is negative $(-12+5.29<0)$. Given the denominator is positive, x_1 is the positive solution.



3. Simplify the positive solution

We simplify x_1 by dividing the numerator and denominator by their common factor, 2:

$$x_1 = \frac{2(2+\sqrt{7})}{2(5+4\sqrt{7})} = \frac{2+\sqrt{7}}{5+4\sqrt{7}}$$

To express the answer in the form $a + b\sqrt{7}$, we **rationalize the denominator** by multiplying the numerator and denominator by the conjugate $5 - 4\sqrt{7}$:

$$\begin{aligned} x_1 &= \frac{(2+\sqrt{7})(5-4\sqrt{7})}{(5+4\sqrt{7})(5-4\sqrt{7})} \\ &= \frac{10 - 8\sqrt{7} + 5\sqrt{7} - 4(7)}{5^2 - (4\sqrt{7})^2} \\ &= \frac{10 - 3\sqrt{7} - 28}{25 - 112} \\ &= \frac{-18 - 3\sqrt{7}}{-87} \\ &= \frac{-3(6+\sqrt{7})}{-87} \\ &= \frac{6+\sqrt{7}}{29} \end{aligned}$$

Separating the terms into the required form:

$$x_1 = \frac{6}{29} + \frac{1}{29}\sqrt{7}$$

$$\boxed{\frac{6}{29} + \frac{1}{29}\sqrt{7}}$$

0606_11_Summer_2020_Q5

Solution

To find the equation of the tangent line to the curve $y = \frac{\ln(3x^2-1)}{x+2}$ at the point where $x = 1$, we follow these steps:

1. Determine the coordinates of the point of tangency We evaluate the function y at $x = 1$:

$$\begin{aligned} y(1) &= \frac{\ln(3(1)^2 - 1)}{1+2} \\ &= \frac{\ln(2)}{3} \end{aligned}$$

Thus, the point of tangency is $\left(1, \frac{\ln(2)}{3}\right)$.

2. Find the derivative of the function To find the slope of the tangent line, we calculate the derivative $\frac{dy}{dx}$ using the **quotient rule**:

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$$

Let $u(x) = \ln(3x^2 - 1)$ and $v(x) = x + 2$. Applying the **chain rule** to $u(x)$:

- $u'(x) = \frac{1}{3x^2-1} \cdot 6x = \frac{6x}{3x^2-1}$
- $v'(x) = 1$

Substituting these into the quotient rule formula:

$$\frac{dy}{dx} = \frac{\left(\frac{6x}{3x^2-1}\right)(x+2) - \ln(3x^2-1)(1)}{(x+2)^2}$$

3. Calculate the slope m at $x = 1$ Evaluate the derivative at $x = 1$:

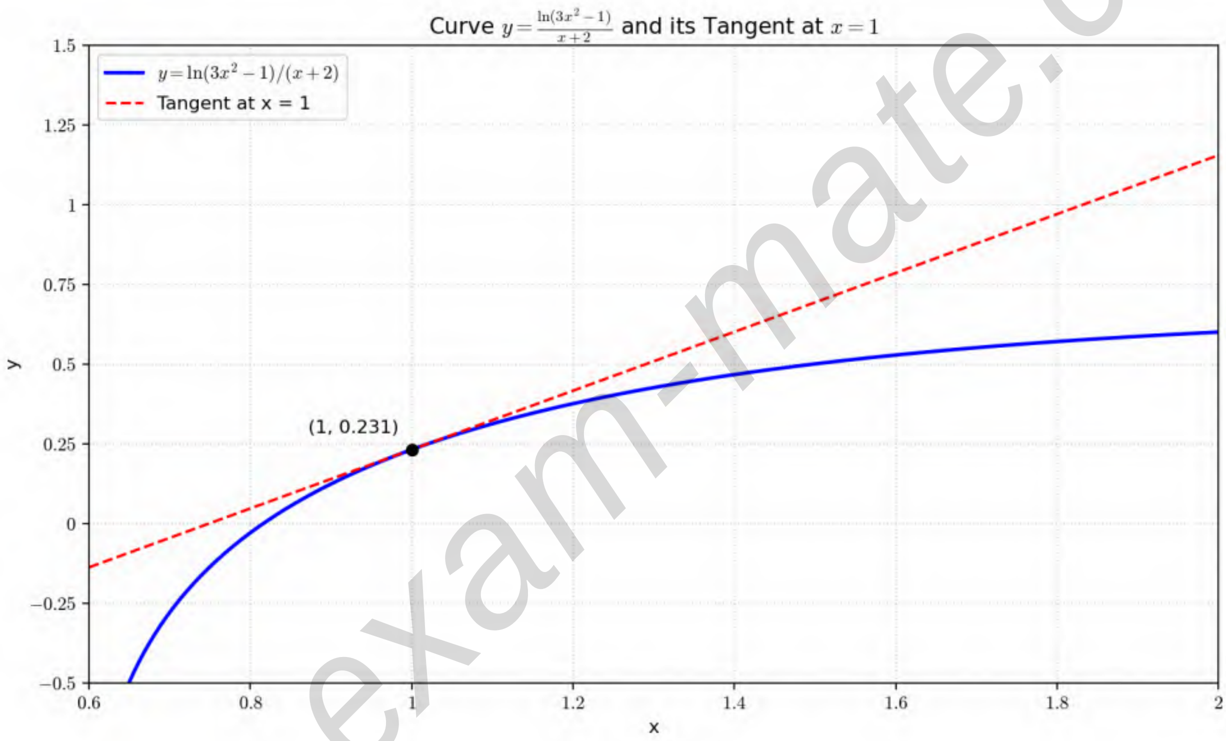
$$\begin{aligned} m &= \frac{dy}{dx}|_{x=1} \\ &= \frac{\left(\frac{6(1)}{3(1)^2-1}\right)(1+2) - \ln(3(1)^2-1)}{(1+2)^2} \\ &= \frac{\left(\frac{6}{2}\right)(3) - \ln(2)}{3^2} \\ &= \frac{9 - \ln(2)}{9} \\ &= 1 - \frac{\ln(2)}{9} \end{aligned}$$

4. Determine the equation of the tangent line Using the point-slope form $y - y_0 = m(x - x_0)$ with $(x_0, y_0) = \left(1, \frac{\ln(2)}{3}\right)$:

$$y - \frac{\ln(2)}{3} = \left(1 - \frac{\ln(2)}{9}\right)(x-1)$$
$$y = \left(1 - \frac{\ln(2)}{9}\right)x - \left(1 - \frac{\ln(2)}{9}\right) + \frac{\ln(2)}{3}$$
$$y = \left(1 - \frac{\ln(2)}{9}\right)x - 1 + \frac{\ln(2)}{9} + \frac{3\ln(2)}{9}$$
$$y = \left(1 - \frac{\ln(2)}{9}\right)x + \left(\frac{4\ln(2)}{9} - 1\right)$$

5. Convert constants to 3 decimal places Using the value $\ln(2) \approx 0.693147$:

- $m = 1 - \frac{0.693147}{9} \approx 1 - 0.077016 = 0.922984 \approx 0.923$
- $c = \frac{4(0.693147)}{9} - 1 \approx 0.308065 - 1 = -0.691935 \approx -0.692$



The equation of the tangent line is:

$$\boxed{y = 0.923x - 0.692}$$

0606_11_Summer_2020_Q6

Solution

1. Finding the coordinates of A and B

To find the intersection points of the line $y = 5x + 6$ and the curve $xy = 8$, we substitute the expression for y from the line equation into the curve equation:

$$x(5x + 6) = 8$$
$$5x^2 + 6x - 8 = 0$$

We solve this quadratic equation using the **quadratic formula**:

$$\begin{aligned} x &= \frac{-6 \pm \sqrt{6^2 - 4(5)(-8)}}{2(5)} \\ &= \frac{-6 \pm \sqrt{36 + 160}}{10} \\ &= \frac{-6 \pm \sqrt{196}}{10} \\ &= \frac{-6 \pm 14}{10} \end{aligned}$$

This yields two values for x:

- $x_1 = \frac{-6+14}{10} = 0.8$
- $x_2 = \frac{-6-14}{10} = -2$

Now, we find the corresponding y-coordinates using $y = 5x + 6$:

- For $x = 0.8$: $y = 5(0.8) + 6 = 4 + 6 = 10$
- For $x = -2$: $y = 5(-2) + 6 = -10 + 6 = -4$

Thus, the coordinates of A and B are:

$$\boxed{A(0.8, 10) \quad \text{and} \quad B(-2, -4)}$$

2. Finding the perpendicular bisector of AB

- **Midpoint of AB (M):**

$$\begin{aligned} M &= \left(\frac{0.8 + (-2)}{2}, \frac{10 + (-4)}{2}\right) \\ &= (-0.6, 3) \end{aligned}$$

- **Gradient of the perpendicular bisector ($m_\perp$):** The gradient of the line AB is $m_{AB} = 5$. The gradient of the **perpendicular bisector** is the negative reciprocal:

$$m_\perp = -\frac{1}{5} = -0.2$$

- **Equation of the perpendicular bisector:** Using the point-slope form $y - y_M = m_\perp(x - x_M)$:

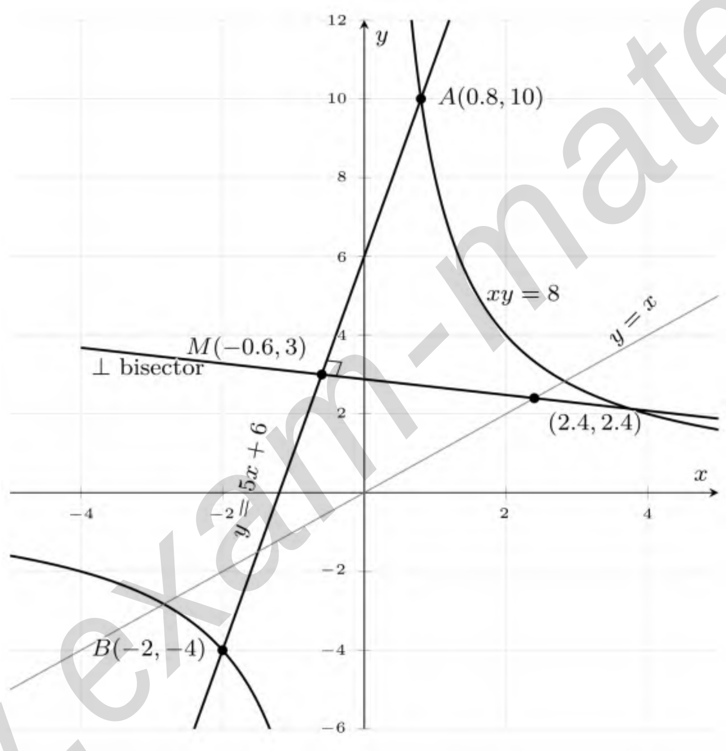
$$y - 3 = -0.2(x - (-0.6))$$
$$y - 3 = -0.2(x + 0.6)$$
$$y = -0.2x - 0.12 + 3$$
$$y = -0.2x + 2.88$$

3. Intersection with the line $y = x$

To find where the perpendicular bisector meets the line $y = x$, we set $y = x$ in the equation derived above:

$$x = -0.2x + 2.88$$
$$1.2x = 2.88$$
$$x = \frac{2.88}{1.2}$$
$$x = 2.4$$

Since the point lies on the line $y = x$, the y-coordinate is also 2.4.



The coordinates of the intersection point are:

$$\boxed{(2.4, 2.4)}$$

0606_11_Summer_2020_Q7

Solution

1. Calculation of the angle θ

The length of an arc s is given by the **arc length formula** $s = r\theta$, where r is the radius and θ is the central angle in radians. Given the arc length $CD = 9.6\,\text{cm}$ and the radius $r = 12\,\text{cm}$:

$$9.6 = 12 \cdot \theta$$
$$\theta = \frac{9.6}{12}$$
$$\theta = 0.8\,\text{rad}$$

2. Calculation of the total area of the shaded regions

The total area of the shaded regions is the difference between the area of the isosceles triangle OAB and the area of the circular sector OCD.

- **Area of sector OCD:** Using the **sector area formula** $A = \frac{1}{2}r^2\theta$:

$$\begin{aligned}\text{Area}_{\text{sector}} &= \frac{1}{2} \cdot (12)^2 \cdot 0.8 \\ &= 0.4 \cdot 144 \\ &= 57.6\,\text{cm}^2\end{aligned}$$

- **Area of triangle OAB:** Since the arc CD touches the line AB at point M, the radius OM is perpendicular to AB. In the isosceles triangle OAB, OM is the altitude and bisects $\angle AOB$. Thus, $\angle AOM = \frac{\theta}{2} = 0.4\,\text{rad}$ and $OM = 12\,\text{cm}$. In the right-angled triangle OAM:

$$\tan(0.4) = \frac{AM}{OM}$$
$$AM = 12\tan(0.4)$$

  The area of $\triangle OAB$ is:

$$\begin{aligned}\text{Area}_{\triangle OAB} &= 2 \cdot \text{Area}_{\triangle OAM} \\ &= 2 \cdot \left(\frac{1}{2} \cdot OM \cdot AM\right) \\ &= 12 \cdot 12\tan(0.4) \\ &= 144\tan(0.4) \approx 60.8822\,\text{cm}^2\end{aligned}$$

- **Total shaded area:**

$$\begin{aligned}\text{Area}_{\text{shaded}} &= 144\tan(0.4) - 57.6 \\ &\approx 60.8822 - 57.6 \\ &\approx 3.2822\,\text{cm}^2\end{aligned}$$

3. Calculation of the total perimeter of the shaded regions

The shaded regions consist of two parts. The perimeter of the left region is $AC + AM + \text{arc } CM$, and the perimeter of the right region is $BD + BM + \text{arc } DM$. By symmetry, the total perimeter P is:

$$P = 2(AC + AM + \text{arc}\ \ CM)$$

- **Length of AC:** In $\triangle OAM$, $\cos(0.4) = \frac{OM}{OA}$, so $OA = \frac{12}{\cos(0.4)}$.

$$AC = OA - OC = \frac{12}{\cos(0.4)} - 12 \approx 13.0284 - 12 = 1.0284\,\text{cm}$$

- **Length of AM:**

$$AM = 12\tan(0.4) \approx 5.0735\,\text{cm}$$

- **Length of arc CM:**

$$\text{arc}\ \ CM = \frac{1}{2}\cdot\text{arc}\ \ CD = \frac{9.6}{2} = 4.8\,\text{cm}$$

- **Total perimeter:**

$$\begin{aligned} P &= 2\left(\frac{12}{\cos(0.4)} - 12 + 12\tan(0.4) + 4.8\right) \\ &= 2\left(\frac{12}{\cos(0.4)} + 12\tan(0.4) - 7.2\right) \\ &\approx 2(1.0284 + 5.0735 + 4.8) \\ &\approx 2(10.9019) \\ &\approx 21.8039\,\text{cm} \end{aligned}$$

(a) $\theta = \boxed{0.8}$
(b) Total area $\approx \boxed{3.28\,\text{cm}^2}$
(c) Total perimeter $\approx \boxed{21.8\,\text{cm}}$

0606_11_Summer_2020_Q8

Solution

1. Algebraic Verification

To show that the sum of the two rational expressions is equivalent to the given fraction, we find a common denominator. The common denominator for $(2x-3)$ and $(2x+3)$ is their product, which is a **difference of squares**:

$$(2x-3)(2x+3) = (2x)^2 - 3^2 = 4x^2 - 9$$

Now, we combine the fractions:

$$\begin{aligned}\frac{3}{2x-3} + \frac{3}{2x+3} &= \frac{3(2x+3) + 3(2x-3)}{(2x-3)(2x+3)} \\ &= \frac{6x+9+6x-9}{4x^2-9} \\ &= \frac{12x}{4x^2-9}\end{aligned}$$

Thus, the identity is verified.

2. Integration

Using the identity from part (a), we can rewrite the integral of the rational function as the integral of two simpler terms:

$$\begin{aligned}\int \frac{12x}{4x^2-9} dx &= \int \left(\frac{3}{2x-3} + \frac{3}{2x+3}\right) dx \\ &= \frac{3}{2} \ln|\ 2x-3\ | + \frac{3}{2} \ln|\ 2x+3\ | + C\end{aligned}$$

To express the result as a **single logarithm**, we apply the **logarithm laws**, specifically $n \ln A = \ln A^n$ and $\ln A + \ln B = \ln(AB)$:

$$\begin{aligned}\frac{3}{2} \ln|\ 2x-3\ | + \frac{3}{2} \ln|\ 2x+3\ | + C &= \frac{3}{2} \ln|\ (2x-3)(2x+3)\ | + C \\ &= \frac{3}{2} \ln|\ 4x^2-9\ | + C \\ &= \ln|\ 4x^2-9\ |^{3/2} + C\end{aligned}$$

The integral is $\ln|\ 4x^2-9\ |^{3/2} + C$.

3. Finding the Exact Value of a

We evaluate the definite integral from 2 to a:

$$\int_2^a \frac{12x}{4x^2-9}dx = \left[\frac{3}{2}\ln|\ 4x^2-9\ |\right]_2^a$$
$$= \frac{3}{2}\ln(4a^2-9) - \frac{3}{2}\ln(4(2)^2-9)$$
$$= \frac{3}{2}\ln(4a^2-9) - \frac{3}{2}\ln(7)$$

We are given that this integral equals $\ln 5\sqrt{5}$. Note that $5\sqrt{5} = 5^{3/2}$, so $\ln 5\sqrt{5} = \frac{3}{2}\ln 5$. Setting the expressions equal:

$$\frac{3}{2}\ln(4a^2-9) - \frac{3}{2}\ln 7 = \frac{3}{2}\ln 5$$
$$\ln(4a^2-9) - \ln 7 = \ln 5$$
$$\ln\left(\frac{4a^2-9}{7}\right) = \ln 5$$

By the **one-to-one property** of the logarithmic function:

$$\frac{4a^2-9}{7} = 5$$
$$4a^2 - 9 = 35$$
$$4a^2 = 44$$
$$a^2 = 11$$

Since the problem states $a > 2$, we take the positive square root:

$$\boxed{a = \sqrt{11}}$$

0606_11_Summer_2020_Q9

Solution

Part (a)

1. **Define the parameters for the** arithmetic progression**.** Let a be the first term and d be the common difference. The n-th term is given by $u_n = a + (n-1)d$ and the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$.
2. **Set up the equations from the given information.**
   - The second term is -14:
     $$a + d = -14 \quad \text{— (1)}$$
   - The sum of the first 21 terms is 84:
     $$S_{21} = \frac{21}{2}[2a + (21-1)d] = 84$$
     $$\frac{21}{2}[2a + 20d] = 84$$
     $$21(a + 10d) = 84$$
     $$a + 10d = 4 \quad \text{— (2)}$$
3. **Solve the system of linear equations.** Subtracting equation (1) from equation (2):
   $$(a + 10d) - (a + d) = 4 - (-14)$$
   $$9d = 18$$
   $$d = 2$$
   Substitute $d = 2$ back into equation (1):
   $$a + 2 = -14$$
   $$a = -16$$
4. **Calculate the 21st term.**
   $$\begin{aligned} u_{21} &= a + 20d \\ &= -16 + 20(2) \\ &= -16 + 40 \\ &= 24 \end{aligned}$$

The first term is -16 and the 21st term is 24.

Part (b)

1. **(i) Find r in terms of p.** In a **geometric progression**, the n-th term is $u_n = ar^{n-1}$.
   - $u_2 = ar = 27p^2$
   - $u_5 = ar^4 = p^5$ Dividing u_5 by u_2:

$$\frac{ar^4}{ar} = \frac{p^5}{27p^2}$$
$$r^3 = \frac{p^3}{27}$$
$$r = \sqrt[3]{\frac{p^3}{27}}$$
$$r = \frac{p}{3}$$

Given $0 < r < 1$, this expression is valid for $0 < p < 3$.

2. **(ii) Find the sum to infinity in terms of p.** First, find the first term a in terms of p:

$$a \cdot \left(\frac{p}{3}\right) = 27p^2$$
$$a = \frac{27p^2 \cdot 3}{p}$$
$$a = 81p$$

The **sum to infinity** is given by $S_\infty = \frac{a}{1-r}$:

$$S_\infty = \frac{81p}{1 - \frac{p}{3}}$$
$$= \frac{81p}{\frac{3-p}{3}}$$
$$= \frac{243p}{3-p}$$

3. **(iii) Find the value of p given $S_\infty = 81$.**

$$\frac{243p}{3-p} = 81$$
$$243p = 81(3-p)$$
$$3p = 3 - p$$
$$4p = 3$$
$$p = \frac{3}{4}$$

$$\boxed{p = \frac{3}{4}}$$

0606_11_Summer_2020_Q10

Solution

Part (a)

(i) Proof of the Identity

To show that $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2\cot^2\theta$, we begin by combining the terms on the left-hand side (LHS) over a common denominator:

$$\begin{aligned}\text{LHS} &= \frac{(\sec\theta + 1) - (\sec\theta - 1)}{(\sec\theta - 1)(\sec\theta + 1)} \\ &= \frac{\sec\theta + 1 - \sec\theta + 1}{\sec^2\theta - 1} \\ &= \frac{2}{\sec^2\theta - 1}\end{aligned}$$

Using the **Pythagorean identity** $1 + \tan^2\theta = \sec^2\theta$, we substitute $\sec^2\theta - 1 = \tan^2\theta$:

$$\begin{aligned}\text{LHS} &= \frac{2}{\tan^2\theta} \\ &= 2\left(\frac{1}{\tan^2\theta}\right) \\ &= 2\cot^2\theta = \text{RHS}\end{aligned}$$

Thus, the identity is proven.

(ii) Solving the Equation

Given the equation $\frac{1}{\sec 2x - 1} - \frac{1}{\sec 2x + 1} = 6$ for $-90^\circ < x < 90^\circ$, we utilize the identity derived in part (i) by substituting $\theta = 2x$:

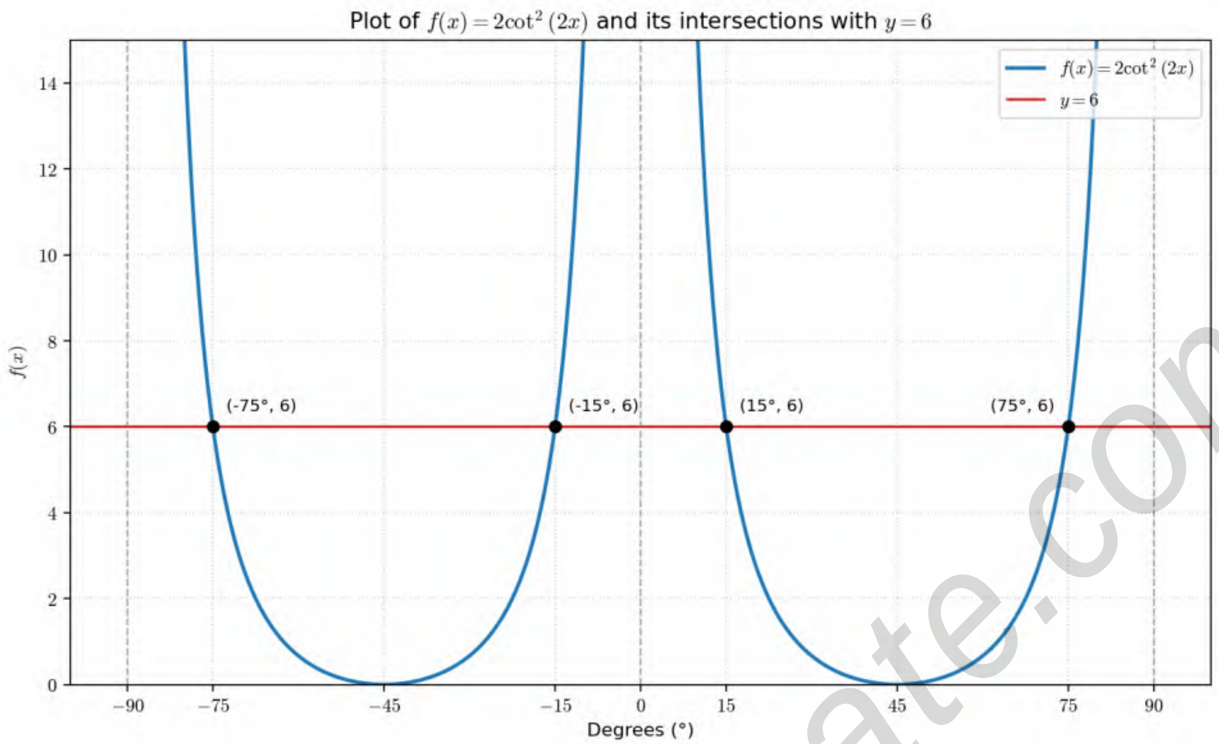
$$\begin{aligned}2\cot^2 2x &= 6 \\ \cot^2 2x &= 3 \\ \tan^2 2x &= \frac{1}{3} \\ \tan 2x &= \pm\frac{1}{\sqrt{3}}\end{aligned}$$

The range for x is $-90^\circ < x < 90^\circ$, which implies the range for $2x$ is $-180^\circ < 2x < 180^\circ$. We solve for $2x$ within this interval:

- For $\tan 2x = \frac{1}{\sqrt{3}}$: $2x = 30^\circ$ or $2x = -150^\circ$.
- For $\tan 2x = -\frac{1}{\sqrt{3}}$: $2x = -30^\circ$ or $2x = 150^\circ$.

Dividing by 2, we find the values for x:

$$x = 15^\circ, -75^\circ, -15^\circ, 75^\circ$$



$$\boxed{x = -75^\circ, -15^\circ, 15^\circ, 75^\circ}$$

Part (b)

We are asked to solve $\csc(y + \frac{\pi}{3}) = 2$ for $0 \leq y \leq 2\pi$ radians. Recalling that $\csc\theta = \frac{1}{\sin\theta}$, the equation becomes:

$$\frac{1}{\sin(y + \frac{\pi}{3})} = 2$$

$$\sin\left(y + \frac{\pi}{3}\right) = \frac{1}{2}$$

Let $u = y + \frac{\pi}{3}$. Since $0 \leq y \leq 2\pi$, the range for u is:

$$\frac{\pi}{3} \leq u \leq 2\pi + \frac{\pi}{3} \implies \frac{\pi}{3} \leq u \leq \frac{7\pi}{3}$$

The general solutions for $\sin u = \frac{1}{2}$ are $u = \frac{\pi}{6} + 2k\pi$ and $u = \frac{5\pi}{6} + 2k\pi$ for $k \in \mathbb{Z}$. We identify the values of u within the specified range:

- $u = \frac{5\pi}{6}$ (since $\frac{\pi}{3} < \frac{5\pi}{6} < \frac{7\pi}{3}$)
- $u = \frac{13\pi}{6}$ (since $\frac{\pi}{3} < \frac{13\pi}{6} < \frac{7\pi}{3}$)

Now, solve for y:

1. $y + \frac{\pi}{3} = \frac{5\pi}{6} \implies y = \frac{5\pi}{6} - \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
2. $y + \frac{\pi}{3} = \frac{13\pi}{6} \implies y = \frac{13\pi}{6} - \frac{2\pi}{6} = \frac{11\pi}{6}$

Both values are within the interval $[0, 2\pi]$.

$$\boxed{y = \frac{\pi}{2}, \frac{11\pi}{6}}$$

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Solution

To find the equation of the curve, we must perform successive **integration** of the given second-order **differential equation** and apply the provided boundary conditions to determine the constants of integration.

1. Find the expression for the gradient The gradient of the curve is given by the first derivative $\frac{dy}{dx}$. We integrate the second derivative with respect to x:

$$\begin{aligned}\frac{dy}{dx} &= \int \frac{d^2y}{dx^2}\,dx \\ &= \int 5\cos 2x\,dx \\ &= \frac{5}{2}\sin 2x + C_1\end{aligned}$$

2. Determine the first constant of integration We are given that the gradient is $\frac{3}{4}$ at the point $(-\frac{\pi}{12}, \frac{5\pi}{4})$. Substituting $x = -\frac{\pi}{12}$ and $\frac{dy}{dx} = \frac{3}{4}$:

$$\begin{aligned}\frac{3}{4} &= \frac{5}{2}\sin\left(2\cdot\left(-\frac{\pi}{12}\right)\right) + C_1 \\ \frac{3}{4} &= \frac{5}{2}\sin\left(-\frac{\pi}{6}\right) + C_1 \\ \frac{3}{4} &= \frac{5}{2}\left(-\frac{1}{2}\right) + C_1 \\ \frac{3}{4} &= -\frac{5}{4} + C_1 \\ C_1 &= \frac{3}{4} + \frac{5}{4} = 2\end{aligned}$$

Thus, the expression for the gradient is:

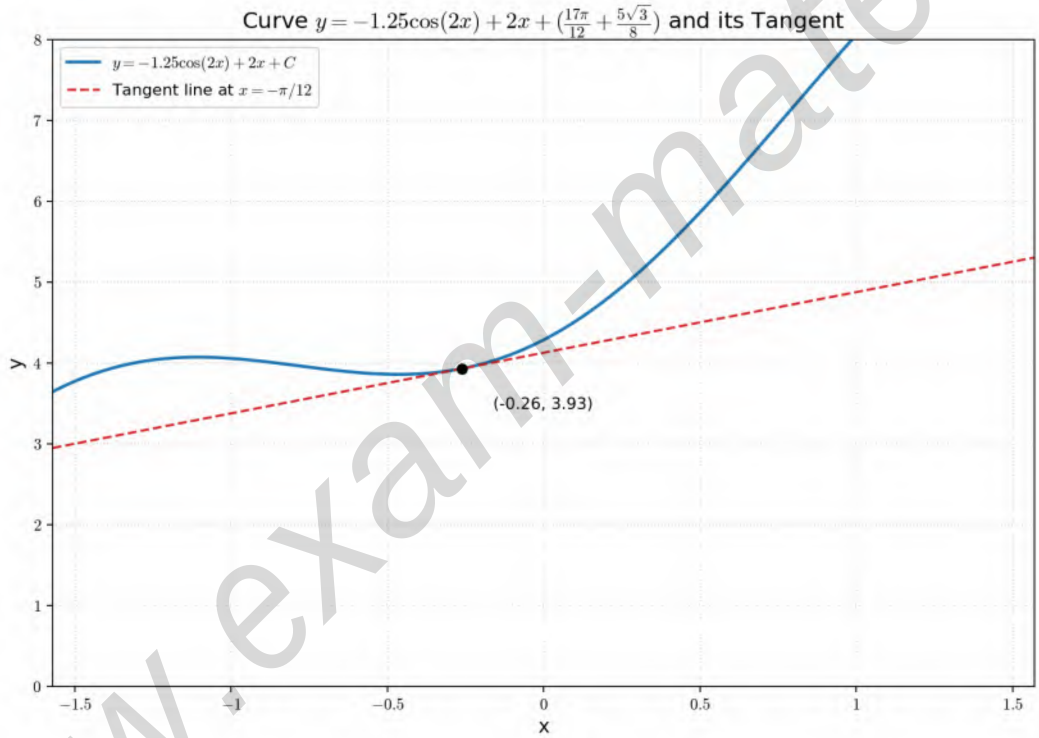
$$\frac{dy}{dx} = \frac{5}{2}\sin 2x + 2$$

3. Find the equation of the curve We integrate the gradient expression to find y:

$$\begin{aligned}y &= \int\left(\frac{5}{2}\sin 2x + 2\right)dx \\ &= -\frac{5}{4}\cos 2x + 2x + C_2\end{aligned}$$

4. Determine the second constant of integration The curve passes through the point $(-\frac{\pi}{12}, \frac{5\pi}{4})$. Substituting these coordinates into the equation:

$$\frac{5\pi}{4} = -\frac{5}{4}\cos\left(2\cdot\left(-\frac{\pi}{12}\right)\right) + 2\left(-\frac{\pi}{12}\right) + C_2$$
$$\frac{5\pi}{4} = -\frac{5}{4}\cos\left(-\frac{\pi}{6}\right) - \frac{\pi}{6} + C_2$$
$$\frac{5\pi}{4} = -\frac{5}{4}\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{6} + C_2$$
$$\frac{5\pi}{4} = -\frac{5\sqrt{3}}{8} - \frac{\pi}{6} + C_2$$
$$C_2 = \frac{5\pi}{4} + \frac{\pi}{6} + \frac{5\sqrt{3}}{8}$$
$$C_2 = \frac{15\pi + 2\pi}{12} + \frac{5\sqrt{3}}{8}$$
$$C_2 = \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$$



5. Final Equation Substituting C_2 back into the expression for y, we obtain the equation of the curve:

$$y = -\frac{5}{4}\cos 2x + 2x + \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$$

$$\boxed{y = 2x - \frac{5}{4}\cos 2x + \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}}$$